Lecture 3
Vector analysis and calculus

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From David K.Cheng CH2:
Vector Analysis

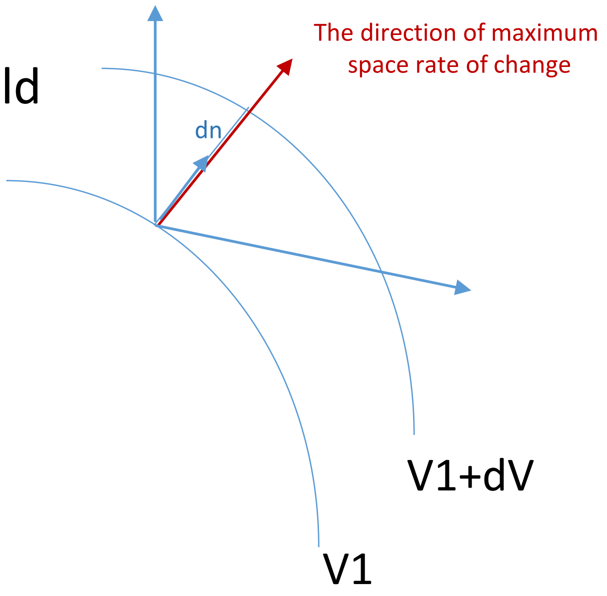
- Gradient of a Scalar Field
- Divergence of a Vector Field
- Divergence Theorem
- Curl of a Vector Field
- Stocke's Theorem

Differential operator ∇ (Del operator):

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z \qquad \nabla = \frac{\partial}{\partial \rho}\hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{a}_\phi + \frac{\partial}{\partial z}\hat{a}_z \qquad \nabla = \frac{\partial}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\hat{a}_\phi$$

Gradient of a Scalar Field (V1)

It is a vector that its magnitude describes maximum rate of change of a scalar field and its direction describes the direction of this maximum rate of change at a given point in a given time.



$$\nabla V = \frac{dV}{dn}\hat{a}_n = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$$

Properties:

$$\nabla(V+U) = \nabla V + \nabla U$$

$$\nabla(VU) = (\nabla V) * U + (\nabla U) * V$$

$$\nabla V^n = nV^{n-1}\nabla V$$

EXAMPLE 2–16 The electrostatic field intensity **E** is derivable as the negative gradient of a scalar electric potential V; that is, $\mathbf{E} = -\nabla V$. Determine **E** at the point (1, 1, 0) if

a) $V = V_0 e^{-x} \sin \frac{\pi y}{4}$,

Solution

$$\text{a) } \mathbf{E} = -\left[\mathbf{a}_x \frac{\partial}{\partial x} + \mathbf{a}_y \frac{\partial}{\partial y} + \mathbf{a}_z \frac{\partial}{\partial z}\right] E_0 e^{-x} \sin \frac{\pi y}{4}$$

$$= \left(\mathbf{a}_x \sin \frac{\pi y}{4} - \mathbf{a}_y \frac{\pi}{4} \cos \frac{\pi y}{4}\right) E_0 e^{-x}.$$

Thus, $\mathbf{E}(1, 1, 0) = \left(\mathbf{a}_x - \mathbf{a}_y \frac{\pi}{4}\right) \frac{E_0}{\sqrt{2}} = \mathbf{a}_E E,$

where

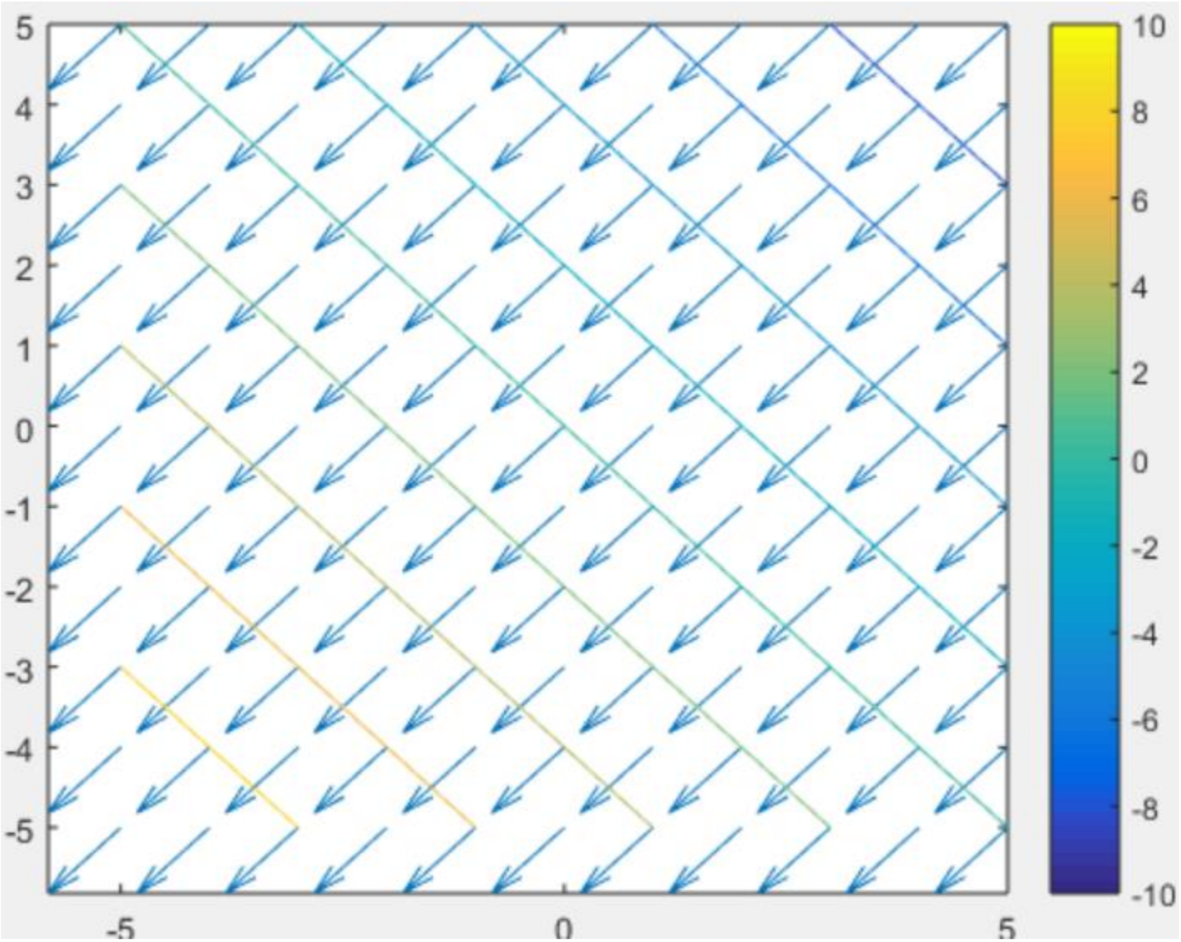
$$E = E_0 \sqrt{\frac{1}{2}\left(1 + \frac{\pi^2}{16}\right)},$$

$$\mathbf{a}_E = \frac{1}{\sqrt{1 + (\pi^2/16)}} \left(\mathbf{a}_x - \mathbf{a}_y \frac{\pi}{4}\right).$$

Gradient of a Scalar function f(x,y)

example

f=-x-y
∇f=-ax-ay



```
[x,y]=meshgrid(-5:1:5);[Vx,Vy]=gradient(-x-y);quiver(x,y,Vx,Vy)
hold on;[x,y]=meshgrid(linspace(-5,5,50));
z=-x-y;contour(x,y,z);hold off
```

Gradient of a Scalar function f(x,y)

$$f(x,y) = 9 - x^2 - y^2$$

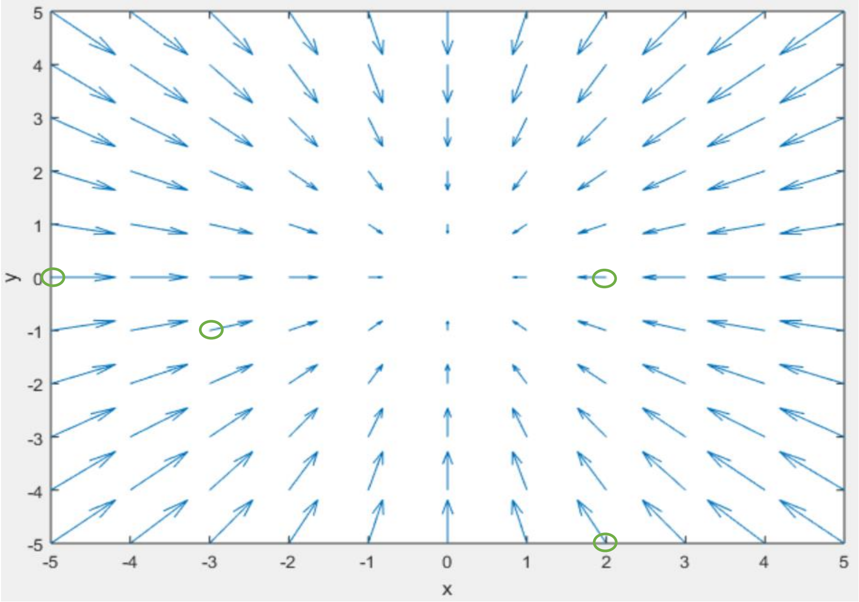
$$\nabla f = -2x\hat{a}_x - 2y\hat{a}_y$$

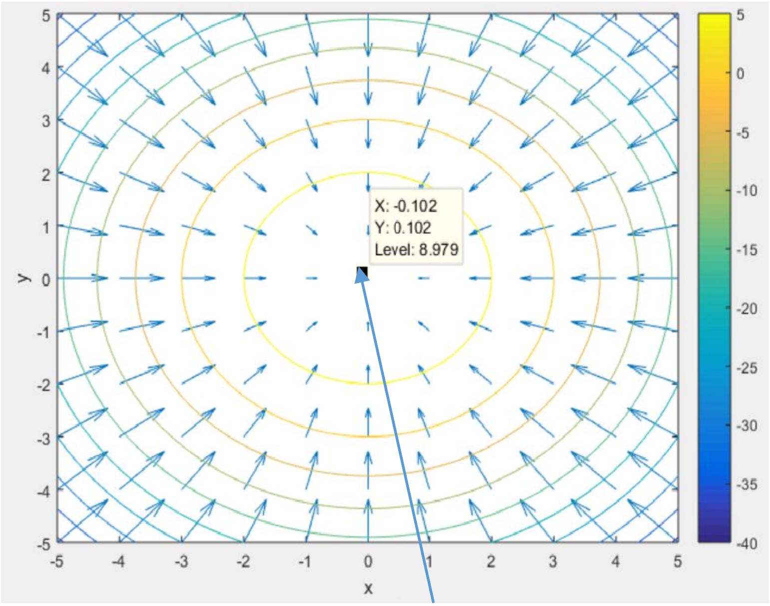
$$at\ (2,-5) \rightarrow \nabla f = -4\hat{a}_x + 10\hat{a}_y = 10.7\angle 111.8^\circ$$

$$(-3,-1) \rightarrow \nabla f = 6\hat{a}_x + 2\hat{a}_y = 7.2\angle 18.4^\circ$$

$$(-5,0) \rightarrow \nabla f = 10\hat{a}_x = 10\angle 0^\circ$$

$$(2,0) \rightarrow \nabla f = -4\hat{a}_x = 4\angle 180^\circ$$





$$f(x,y) = 9 - x^2 - y^2$$
$$= 9 - 0.102^2 - 0.102^2 = 8.979$$

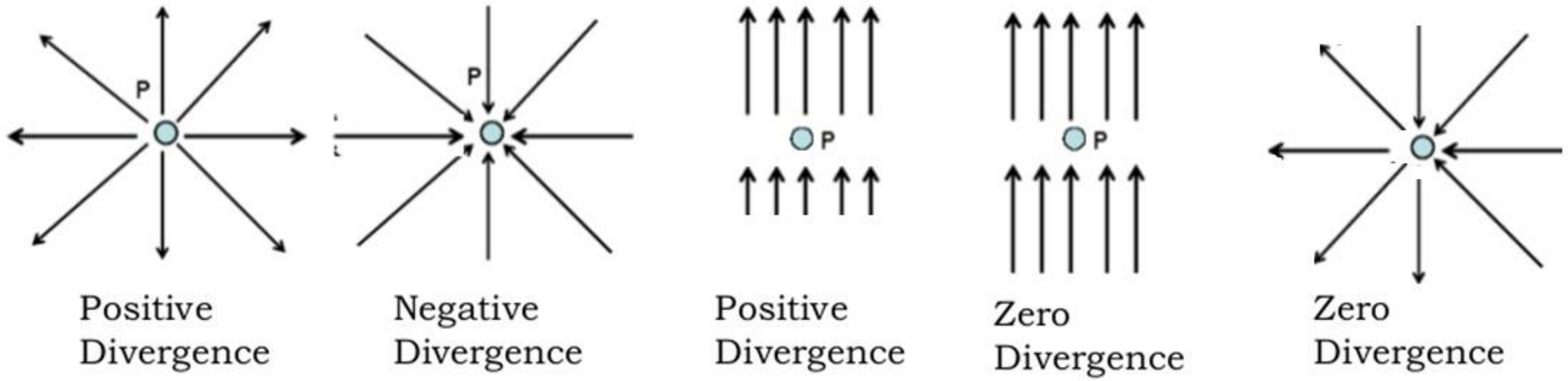
```
[x,y]=meshgrid(-5:1:5);[Vx,Vy]=gradient(9-x.^2-y.^2);quiver(x,y,Vx,Vy)
hold on;[x,y]=meshgrid(linspace(-5,5,50));
z=9-x.^2-y.^2;contour(x,y,z);hold off
```

Divergence of a Vector Field

Divergence of a vector field A is the net outward flux of A per unit volume as the volume about the point tends to zero

$$div\vec{A} \approx \lim_{\Delta v \to 0} \frac{\oint_s \vec{A}.d\vec{s}}{\Delta v}$$

A divergence applied to a vector and result a scalar value that indicate the amount of vector field A(r) that is **Converging to**, or **diverging from** ,a given point.



source of flux at point p

sink of flux at point p

source of flux at point p

Uniform field (solenoid field) or divergenceless field

Nor source nor sink of flux at point p

Divergence of a Vector Field

If $div\ \overline{D} = \lim_{\Delta v \to 0} \frac{\oint_s \overline{D}.d\overline{s}}{\Delta v}$ prove $div\ \overline{D} = (\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z).(D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z) = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

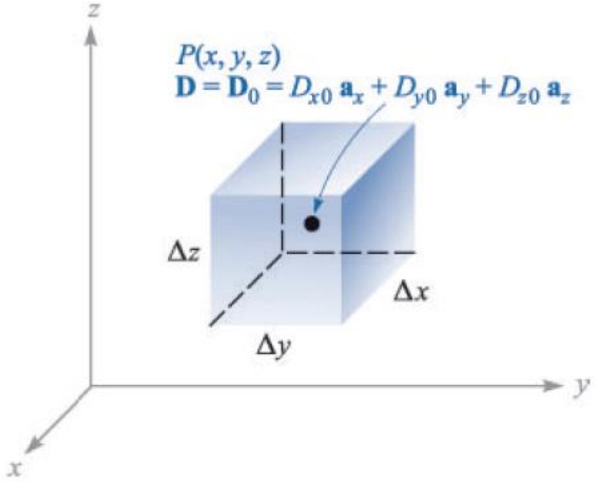
We could use taylor expansion to get D at six faces of the cube

This term vanishes as Δv reach 0 i.e x very close to a

$$f(x) = f(a) + f'(a) * (x-a) + f''(a) * (x-a)^2 + ...$$

Taylor expansion: get value of function in required position by expansion of function in known position + derivative (changes Of function value from known position to desired position multiplied by distance between two positions

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_{\text{front}} + \int_{\text{back}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}}$$



$$\int_{\text{front}} \doteq \mathbf{D}_{\text{front}} \cdot \Delta\mathbf{S}_{\text{front}}$$

$$\doteq \mathbf{D}_{\text{front}} \cdot \Delta y\, \Delta z\, \mathbf{a}_x$$

$$\doteq D_{x,\text{front}} \Delta y\, \Delta z$$

$$D_{x,\text{front}} \doteq D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x}\Big|_{x_0}$$ (using taylor expansion)

$$\int_{\text{front}} \doteq \left(D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\, \Delta z$$

$$\int_{\text{back}} \doteq \mathbf{D}_{\text{back}} \cdot \Delta\mathbf{S}_{\text{back}}$$

$$\doteq \mathbf{D}_{\text{back}} \cdot (-\Delta y\, \Delta z\, \mathbf{a}_x)$$

$$\doteq -D_{x,\text{back}} \Delta y\, \Delta z$$

$$D_{x,\text{back}} \doteq D_{x0} - \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}$$

$$\int_{\text{back}} \doteq \left(-D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\, \Delta z$$

$$\int_{\text{front}} + \int_{\text{back}} \doteq \frac{\partial D_x}{\partial x}\Delta x\, \Delta y\, \Delta z$$

similarly

$$\int_{\text{right}} + \int_{\text{left}} \doteq \frac{\partial D_y}{\partial y}\Delta x\, \Delta y\, \Delta z$$

$$\int_{\text{top}} + \int_{\text{bottom}} \doteq \frac{\partial D_z}{\partial z}\Delta x\, \Delta y\, \Delta z$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} \doteq \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x\, \Delta y\, \Delta z$$

or

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q \doteq \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta v$$

$$\lim_{\Delta v \to 0} \frac{\oint_S \mathbf{D} \cdot d\mathbf{S}}{\Delta v} = \lim_{\Delta v \to 0} \frac{Q}{\Delta v} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right), \quad \lim_{\Delta v \to 0} \frac{Q}{\Delta v} = \rho_v$$

$$\boxed{\text{div}\, \mathbf{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)}$$

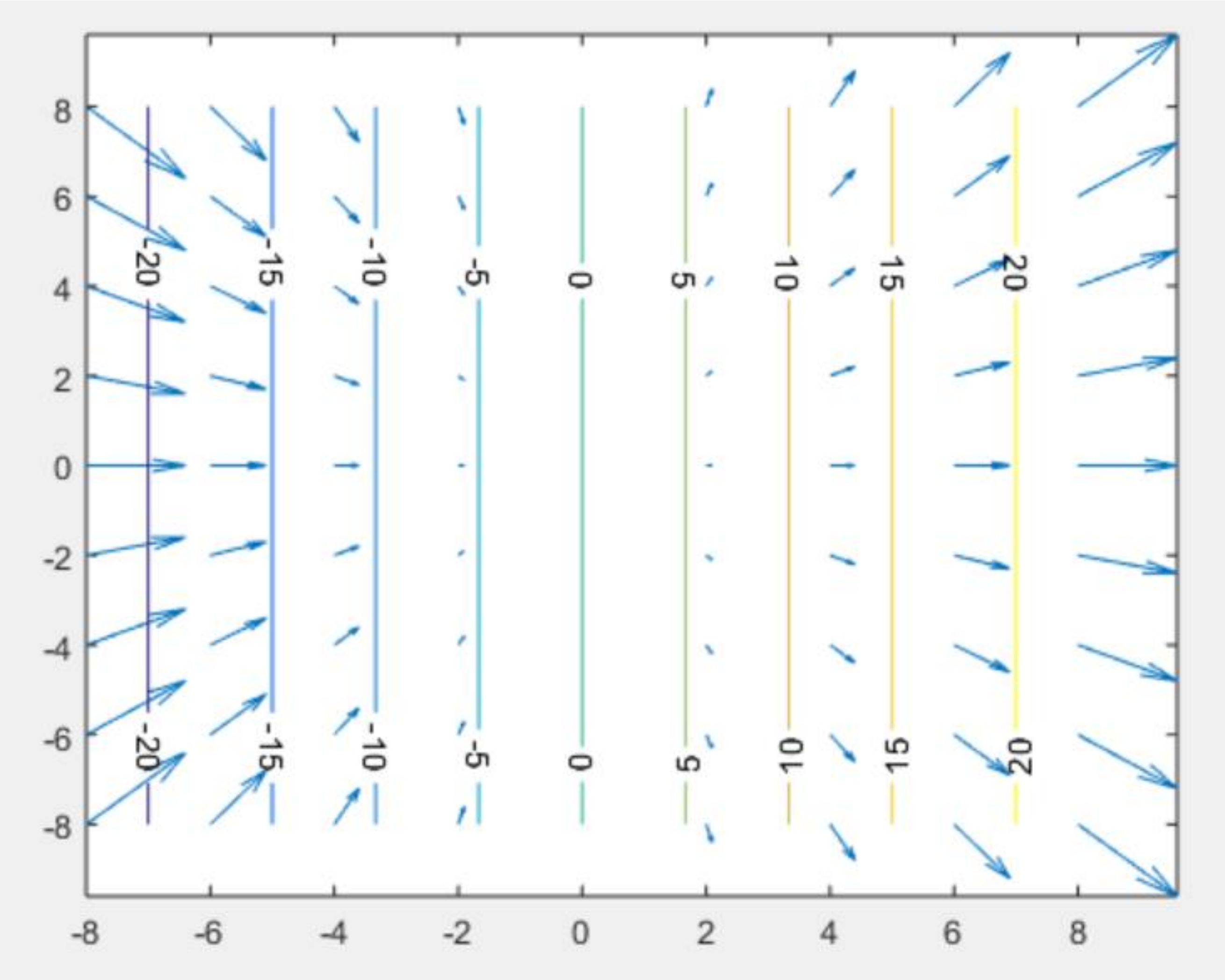
Example:

Given $\overline{A} = x^2 \ \hat{a}_x + xy\ \hat{a}_y$

Find div A, plot A in arrow div A in contour

$$\nabla.\overline{A} = 2x + x = 3x$$

```
[x,y] = meshgrid(-8:2:8,-8:2:8);
Fx = x.^2;
Fy = x.*y;
quiver(x,y,Fx,Fy)
D = divergence(x,y,Fx,Fy);
hold on
contour(x,y,D,'ShowText','on')
hold off
```



EXAMPLE 2–17 Find the divergence of the position vector to an arbitrary point.

Solution We will find the solution in Cartesian as well as in spherical coordinates.

a) *Cartesian coordinates.* The expression for the position vector to an arbitrary point (x, y, z) is

$$\overrightarrow{OP} = \mathbf{a}_x x + \mathbf{a}_y y + \mathbf{a}_z z.$$

$$\nabla \cdot (\overrightarrow{OP}) = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3.$$

b) *Spherical coordinates.* Here the position vector is simply

$$\overrightarrow{OP} = \mathbf{a}_R R.$$

Its divergence in spherical coordinates (R, θ, ϕ) can be obtained from

$$\boxed{\nabla \cdot \mathbf{A} = \frac{1}{R^2}\frac{\partial}{\partial R}(R^2 A_R) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta) + \frac{1}{R\sin\theta}\frac{\partial A_\phi}{\partial\phi}.}$$

$$\nabla \cdot (\overrightarrow{OP}) = 3.$$

Divergence theorem

We define divergence as the net outward flux per unit volume

So the volume integral of the divergence of a vector field equals the total outward flux of the vector through the surface that bounds the volume

$$\int_v \nabla.\vec{A}\, dv = \oint_s \overline{A}.d\overline{s}$$

EXAMPLE 2–19 Given $\mathbf{A} = \mathbf{a}_x x^2 + \mathbf{a}_y xy + \mathbf{a}_z yz$, verify the divergence theorem over a cube one unit on each side. The cube is situated in the first octant of the Cartesian coordinate system with one corner at the origin.

Solution Refer to Fig. 2–28. We first evaluate the surface integral over the six faces.

1. Front face: $x = 1$, $d\mathbf{s} = \mathbf{a}_x\,dy\,dz$;

$$\int_{\substack{\text{front}\\\text{face}}} \mathbf{A}\cdot d\mathbf{s} = \int_0^1\int_0^1 dy\,dz = 1.$$

2. Back face: $x = 0$, $d\mathbf{s} = -\mathbf{a}_x\,dy\,dz$;

$$\int_{\substack{\text{back}\\\text{face}}} \mathbf{A}\cdot d\mathbf{s} = 0.$$

3. Left face: $y = 0$, $d\mathbf{s} = -\mathbf{a}_y\,dx\,dz$;

$$\int_{\substack{\text{left}\\\text{face}}} \mathbf{A}\cdot d\mathbf{s} = 0.$$

4. Right face: $y = 1$, $d\mathbf{s} = \mathbf{a}_y\,dx\,dz$;

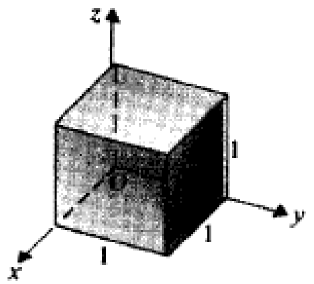
$$\int_{\substack{\text{right}\\\text{face}}} \mathbf{A}\cdot d\mathbf{s} = \int_0^1\int_0^1 x\,dx\,dz = \tfrac{1}{2}.$$

5. Top face: $z = 1$, $d\mathbf{s} = \mathbf{a}_z\,dx\,dy$;

$$\int_{\substack{\text{top}\\\text{face}}} \mathbf{A}\cdot d\mathbf{s} = \int_0^1\int_0^1 y\,dx\,dy = \tfrac{1}{2}.$$

6. Bottom face: $z = 0$, $d\mathbf{s} = -\mathbf{a}_z\,dx\,dy$;

$$\int_{\substack{\text{bottom}\\\text{face}}} \mathbf{A}\cdot d\mathbf{s} = 0.$$



$$\mathbf{A} = \mathbf{a}_x x^2 + \mathbf{a}_y xy + \mathbf{a}_z yz,$$

Adding the above six values, we have

$$\oint_S \mathbf{A}\cdot d\mathbf{s} = 1 + 0 + 0 + \tfrac{1}{2} + \tfrac{1}{2} + 0 = 2.$$

Now the divergence of $\mathbf{A}$ is

$$\nabla\cdot\mathbf{A} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(yz) = 3x + y.$$

Hence,

$$\int_V \nabla\cdot\mathbf{A}\,dv = \int_0^1\int_0^1\int_0^1 (3x + y)\,dx\,dy\,dz = 2,$$

which is the same as the result of the closed surface integral in (2–120). The divergence theorem is therefore verified.

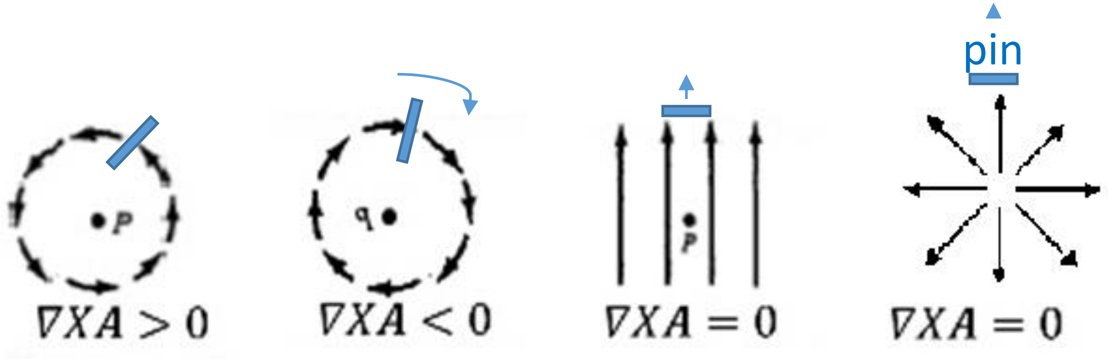
Curl of a Vector Field

Curl of a vector field H ($\nabla\times$H) is a vector (J)whose magnitude is the maximum net circulating of H per unit area as area tends to zero and whose direction is the normal direction of the area (when the area is oriented to make the net circulation Maximum)

Used in ampere law: $\nabla\times$H = J or $\oint \bar{H}\cdot d\bar{l} = I_{enc} = \bar{J}.d\bar{s}$

$$\nabla\times\bar{H} = \lim_{\Delta S_n\to 0}\frac{\oint_c \bar{H}.d\bar{l}}{\Delta S_n}$$

The curl of a vector field A at a point P may indicates how much the field rotate around P or how much field vary In direction normal to its flow direction (if we put pin it cause it to rotate)



$$\oint_c \bar{H}.d\bar{l} = (H.\Delta L)_{1-2} + (H.\Delta L)_{2-3} + (H.\Delta L)_{3-4} + (H.\Delta L)_{4-1}$$

$$(\bar{H}.\Delta\bar{L})_{1-2} = H_{y,1-2}\Delta y = \quad (H_{y0} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x})\Delta y$$

$$(\bar{H}.\Delta\bar{L})_{3-4} = H_{y,3-4}(-\Delta y) = \quad (H_{y0} - \frac{\Delta x}{2}\frac{\partial H_y}{\partial x})(-\Delta y)$$

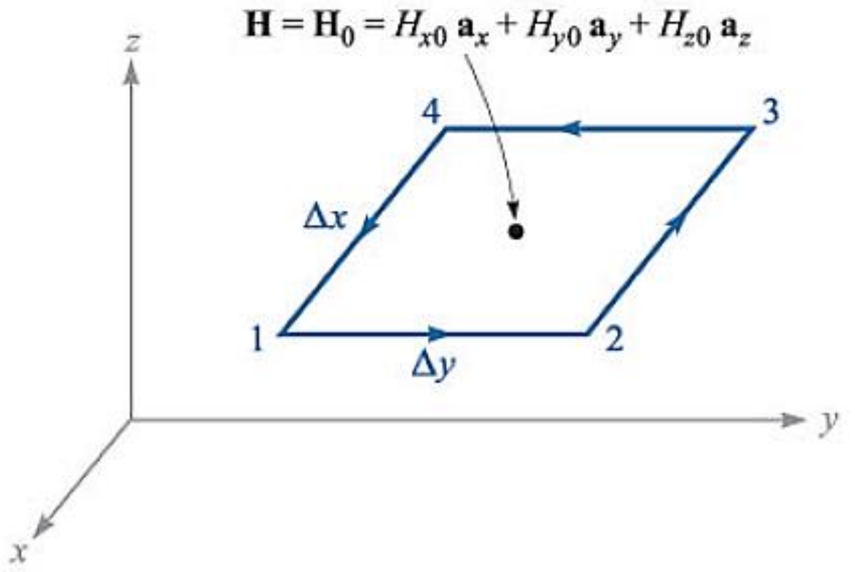
$$(\bar{H}.\Delta\bar{L})_{1-2} + (\bar{H}.\Delta\bar{L})_{3-4} = \Delta x \Delta y \frac{\partial H_y}{\partial x}$$

similarly

$$(\bar{H}.\Delta\bar{L})_{2-3} = H_{x,2-3}(-\Delta x) = -(H_{x0} + \frac{\Delta y}{2}\frac{\partial H_x}{\partial y})\Delta x$$

$$(\bar{H}.\Delta\bar{L})_{4-1} = H_{x,4-1}(\Delta x) = (H_{x0} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y})(\Delta x)$$

$$(\bar{H}.\Delta\bar{L})_{2-3} + (\bar{H}.\Delta\bar{L})_{4-1} = -\Delta x \Delta y \frac{\partial H_x}{\partial y}$$



$$\oint_c \bar{H}.d\bar{l} = \Delta x \Delta y(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}) = J_z \Delta x \Delta y \rightarrow \lim_{\Delta x \Delta y \to 0} \frac{\oint_c \bar{H}.d\bar{l}}{\Delta x \Delta y} = J_z$$

Choosing closed paths at yz plane and xz plane and do analogous processes lead to expressions for the y and z components Of the current density

$$\lim_{\Delta y, \Delta z \to 0} \frac{\oint \mathbf{H} \cdot d\mathbf{L}}{\Delta y \Delta z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

$$\lim_{\Delta z, \Delta x \to 0} \frac{\oint \mathbf{H} \cdot d\mathbf{L}}{\Delta z \Delta x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

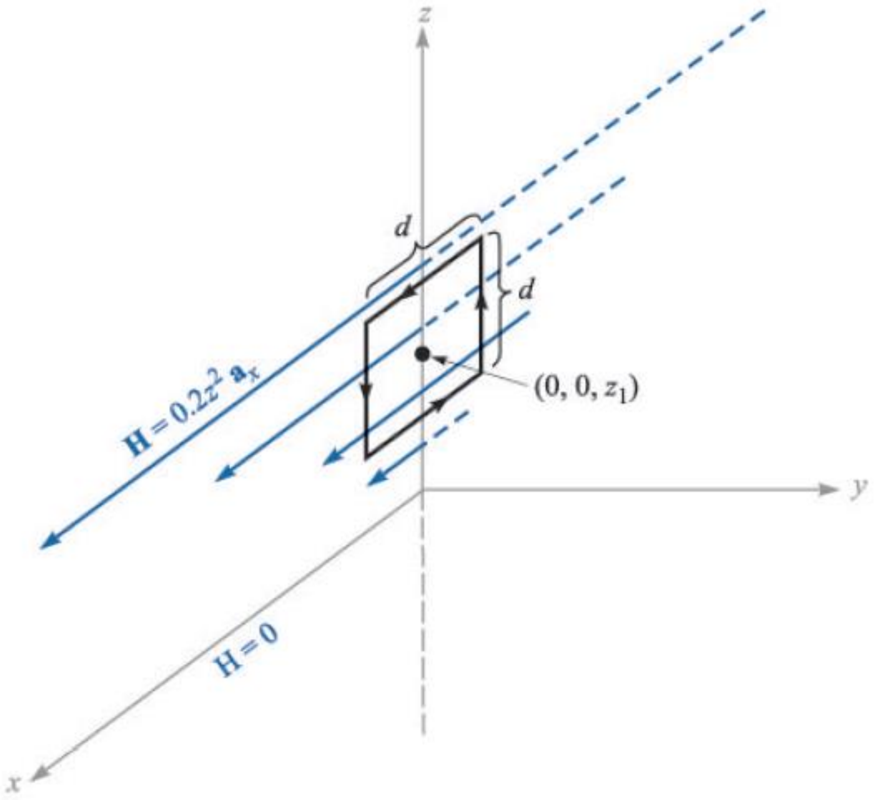
$$\lim_{\Delta S_N \to 0} \frac{\oint \mathbf{H} \cdot d\mathbf{L}}{\Delta S_N} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$$

$$\text{curl } \mathbf{H} = \nabla \times \mathbf{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z = \mathbf{J}$$

$$\boxed{\nabla \times \mathbf{H} = \mathbf{J}}$$ *Ampère's circuital law.*

Example 8.2

As an example of the evaluation of curl **H** from the definition and of the evaluation of another line integral, let us suppose that $\mathbf{H} = 0.2z^2\mathbf{a}_x$ for $z > 0$, and $\mathbf{H} = 0$ elsewhere, as shown in Fig. 8.15. Calculate $\oint \mathbf{H} \cdot d\mathbf{L}$ about a square path with side d, centered at $(0, 0, z_1)$ in the $y = 0$ plane where $z_1 > 2d$.



Solution. We evaluate the line integral of **H** along the four segments, beginning at the top:

$$\oint \mathbf{H} \cdot d\mathbf{L} = 0.2(z_1 + \tfrac{1}{2}d)^2 d + 0 - 0.2(z_1 - \tfrac{1}{2}d)^2 d + 0$$

$$= 0.4 z_1 d^2$$

$$or = dxdz \frac{\partial Hx}{\partial z}$$

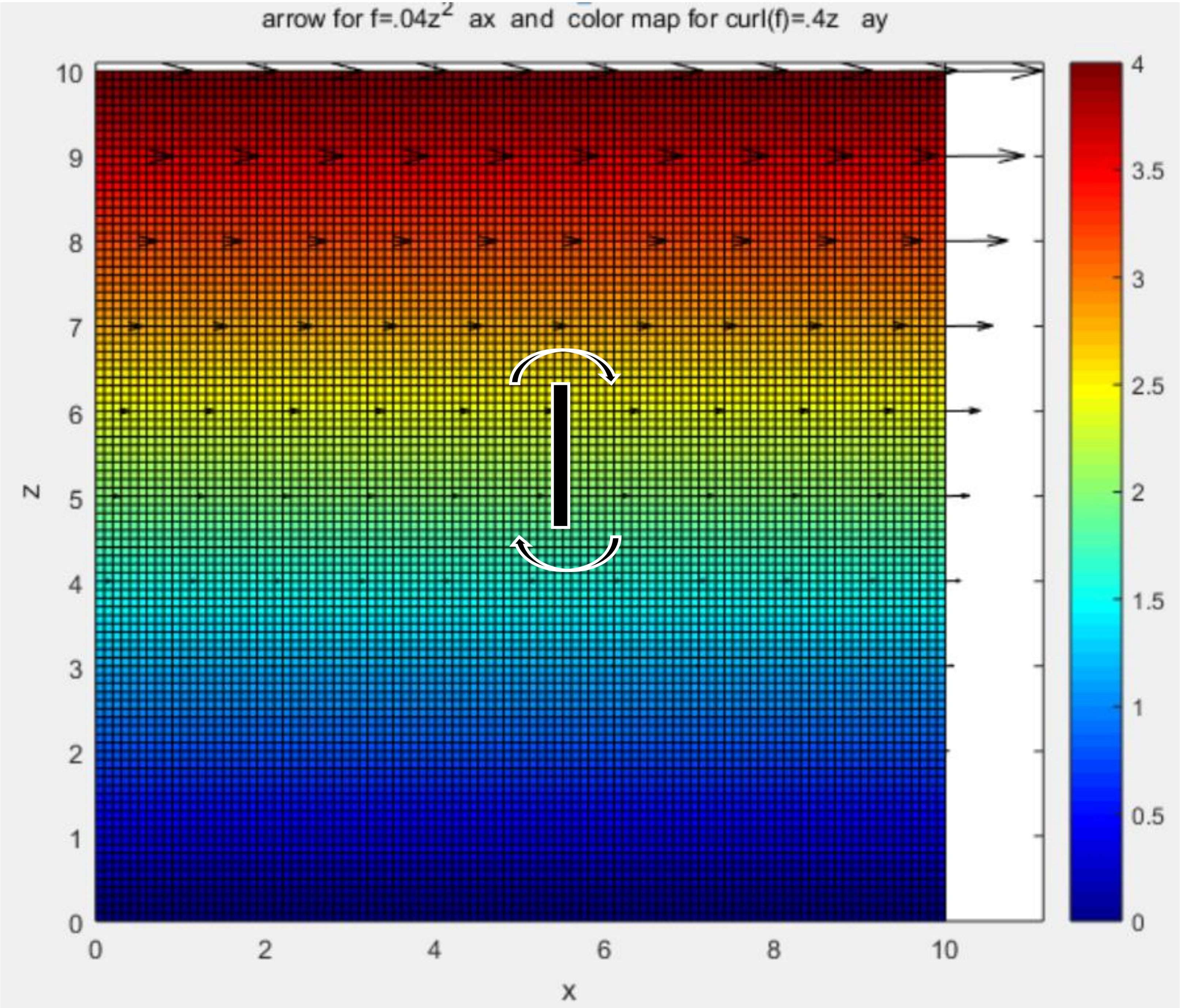
$$as \quad \nabla \times \bar{H} = \lim_{\Delta S_n \to 0} \frac{\oint_c \bar{H}.d\bar{l}}{\Delta S_n}$$

In the limit as the area approaches zero, we find

$$(\nabla \times \mathbf{H})_y = \lim_{d \to 0} \frac{\oint \mathbf{H} \cdot d\mathbf{L}}{d^2} = \lim_{d \to 0} \frac{0.4 z_1 d^2}{d^2} = 0.4 z_1$$

$$\nabla \times \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0.2z^2 & 0 & 0 \end{vmatrix} = \frac{\partial}{\partial z}(0.2z^2)\mathbf{a}_y = 0.4z\mathbf{a}_y$$

which checks with the result above when $z = z_1$.



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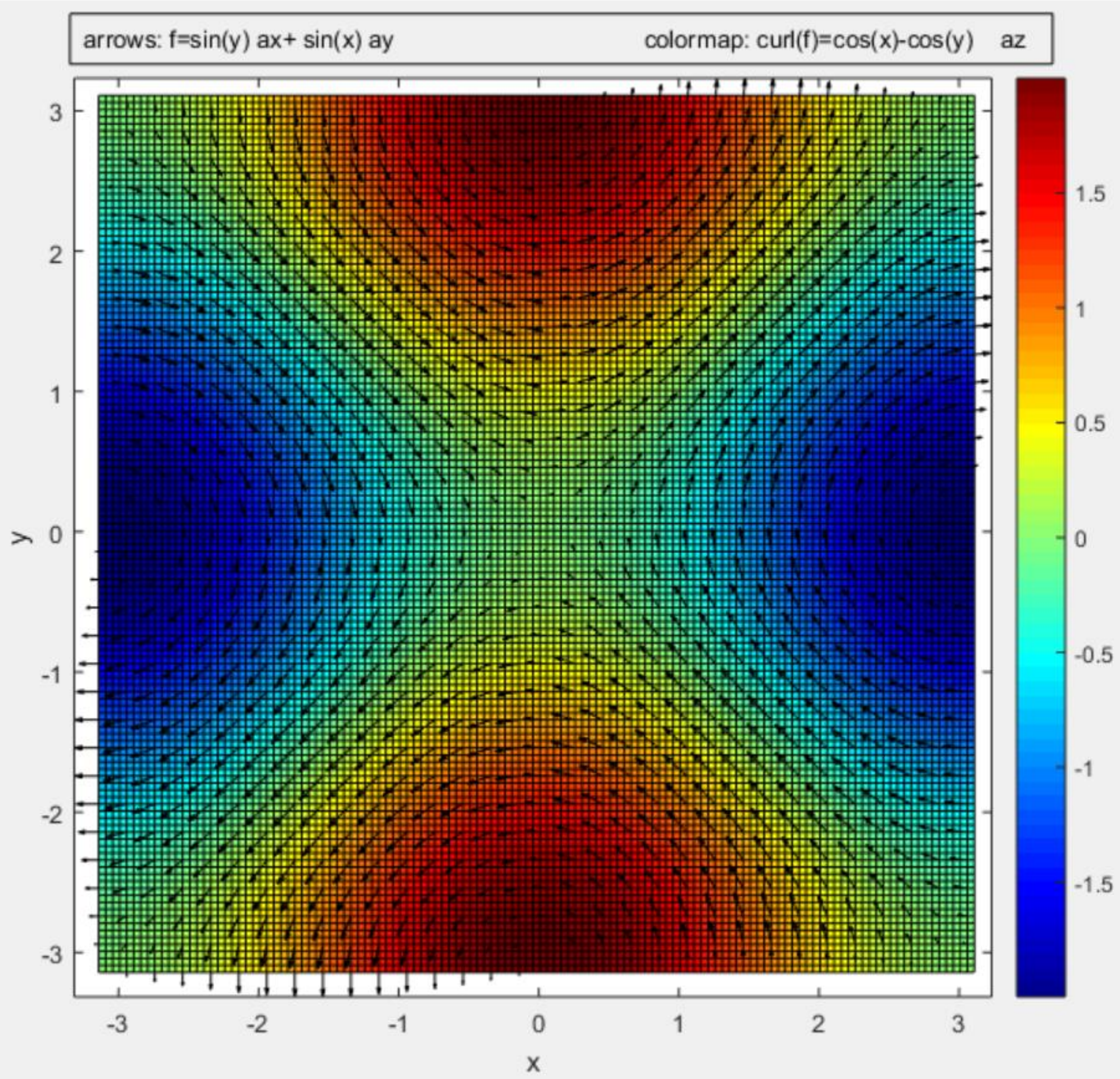
Another example

$$\vec{f} = \sin(y)\hat{a}_x + \sin(x)\hat{a}_y$$

$$\nabla \times \vec{f} = \frac{\partial}{\partial x}(\sin(x)) - \frac{\partial}{\partial y}(\sin(y)) \quad \hat{a}_z$$

$$= \cos(x) - \cos(y) \quad \hat{a}_z$$



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Stocks' Theorem

from Curl definition

$$\nabla \times \overline{H} = \lim_{\Delta S_n \to 0} \frac{\oint_c \overline{H}.d\bar{l}}{\Delta S_n} \rightarrow \lim_{\Delta S_n \to 0} \nabla \times \overline{H}.\Delta \overline{S}_n = \lim_{\Delta S_n \to 0} \oint_c \overline{H}.d\bar{l} \rightarrow \int_s \nabla \times \overline{H}.d\bar{s} = \oint_c \overline{H}.d\bar{l}$$

$$\lim_{\Delta s_j \to 0} \sum_{j=1}^{N} \left(\oint_{c_j} \overline{H}.d\bar{l}\right) = \oint_c \overline{H}.d\bar{l}$$

$$\lim_{\Delta s_j \to 0} \sum_{j=1}^{N} (\nabla \times \overline{H})_j \Delta \bar{s}_j = \int_s \nabla \times \overline{H}.d\bar{s}$$

Stocks' Theorem

$$\int_s \nabla \times \overline{H}.d\bar{s} = \oint_c \overline{H}.d\bar{l}$$

Stocks' Theorem:

The surface integral of the curl of a vector field over an open surface is equal to the closed line integral of the vector along the contour bounding the surface

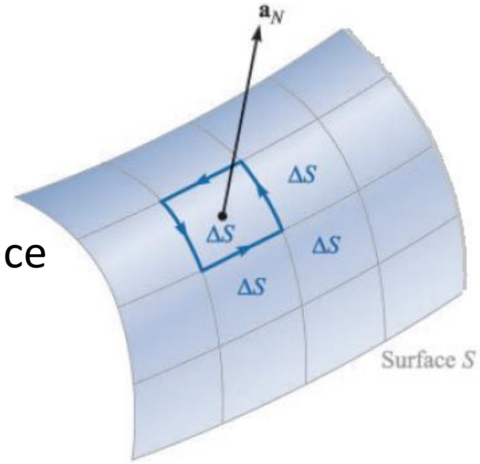

FIGURE 8.16
The sum of the closed line integrals about the perimeter of every ΔS is the same as the closed line integral about the perimeter of S because of cancellation on every interior path.

Note:

$$\oint_s \nabla \times \overline{H}.d\bar{s} = 0$$

For any closed surface no surface bounding contour c exist so curl over any closed surface=0

Example

A numerical example may help to illustrate the geometry involved in Stokes' theorem. Consider the portion of a sphere shown in Fig. 8.17. The surface is specified by $r = 4$, $0 \le \theta \le 0.1\pi$, $0 \le \phi \le 0.3\pi$, and the closed path forming its perimeter is composed of three circular arcs. We are given the field $\mathbf{H} = 6r\sin\phi\,\mathbf{a}_r + 18r\sin\theta\cos\phi\,\mathbf{a}_\phi$ and are asked to evaluate each side of Stokes' theorem.

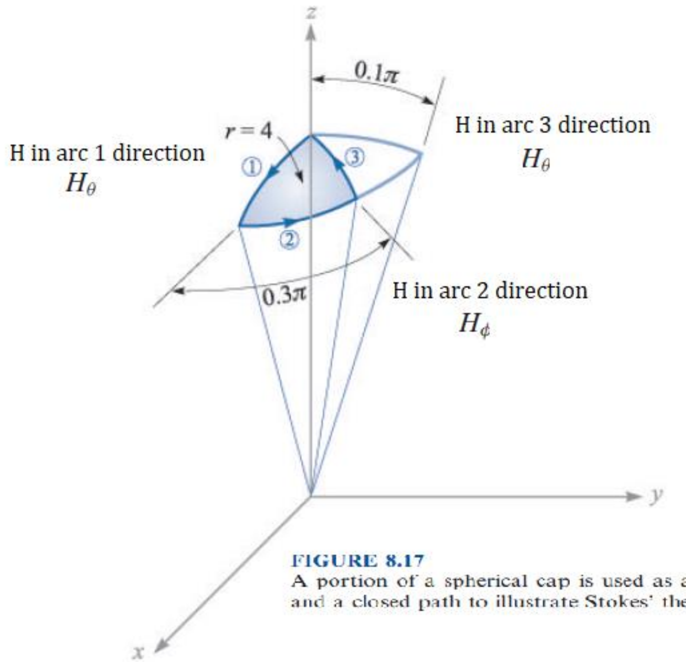

FIGURE 8.17
A portion of a spherical cap is used as a surface and a closed path to illustrate Stokes' theorem.

Solution. The first path segment is described in spherical coordinates by $r = 4$, $0 \le \theta \le 0.1\pi$, $\phi = 0$; the second one by $r = 4$, $\theta = 0.1\pi$, $0 \le \phi \le 0.3\pi$; and the third by $r = 4$, $0 \le \theta \le 0.1\pi$, $\phi = 0.3\pi$. $\quad d\mathbf{L} = dr\,\mathbf{a}_r + r\,d\theta\,\mathbf{a}_\theta + r\sin\theta\,d\phi\,\mathbf{a}_\phi$

$H_\theta = 0$,

0

$$\oint \mathbf{H}\cdot d\mathbf{L} = \int_1 H_\theta r\,d\theta + \int_2 H_\phi r\sin\theta\,d\phi + \int_3 H_\theta r\,d\theta$$

$$\oint \mathbf{H}\cdot d\mathbf{L} = \int_0^{0.3\pi} [18(4)\sin 0.1\pi\cos\phi]4\sin 0.1\pi\,d\phi = 288\sin^2 0.1\pi\sin 0.3\pi = 22.2\text{ A}$$

$$\nabla\times\mathbf{H} = \frac{1}{r\sin\theta}(36r\sin\theta\cos\theta\cos\phi)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}6r\cos\phi - 36r\sin\theta\cos\phi\right)\mathbf{a}_\theta$$

Since $d\mathbf{S} = r^2\sin\theta\,d\theta\,d\phi\,\mathbf{a}_r$, the integral is

$$\int_S (\nabla\times\mathbf{H})\cdot d\mathbf{S} = \int_0^{0.3\pi}\int_0^{0.1\pi} (36\cos\theta\cos\phi)16\sin\theta\,d\theta\,d\phi$$

$$= \int_0^{0.3\pi} 576\left(\tfrac{1}{2}\sin^2\theta\right)\Big|_0^{0.1\pi}\cos\phi\,d\phi$$

$$= 288\sin^2 0.1\pi\sin 0.3\pi = 22.2\text{ A}$$

Assignment (Due date 8/11/2020)
-research on: electromagnetic train

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Project:
-electromagnetic breaking system
-RFID Access control Using Arduino
Required:
-literature review
-theory of operation
-design circuit

Site containing different project ideas in various fields:
https://courses.engr.illinois.edu/ece445/projects.asp